

1 Surface integrals of vector fields

Exercise 1 Let Σ be the portion of the plane $2x + y + z = 6$ inside the cylinder $x^2 + y^2 = 4$, oriented upwards. Compute

$$\iint_{\Sigma} \langle z, y, 3 - x \rangle \cdot dS$$

Exercise 2 Let Σ be the portion of the plane $z = 3x + 2y + 1$ between the planes $x = 0$ and $x = 1$ and between the planes $y = 0$ and $y = 2$, oriented upwards. Compute

$$\iint_{\Sigma} \langle y + 2, z - 3, x + 1 \rangle \cdot dS$$

Exercise 3 Let Σ be the triangle with vertices $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$, oriented downwards. Compute

$$\iint_{\Sigma} \langle x + 2, 2y - 5, 3z + 1 \rangle \cdot dS$$

Exercise 4 Let Σ be the portion of the paraboloid $z = 4 - x^2 - y^2$ above the plane $z = 0$, oriented upwards. Compute

$$\iint_{\Sigma} \langle x, y, 0 \rangle \cdot dS$$

Exercise 5 Let Σ be the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$, oriented outward (away from the axis). Compute

$$\iint_{\Sigma} \langle z, 3 + x, 1 - y \rangle \cdot dS$$

Exercise 6 Let Σ be the portion of the sphere $x^2 + y^2 + z^2 = 9$ with $z \geq 0$, oriented upward. Compute

$$\iint_{\Sigma} \langle x, y, z \rangle \cdot dS$$

Exercise 7 Let Σ be the portion of the paraboloid $y = x^2 + z^2$ inside the cylinder $x^2 + z^2 = 1$, oriented in the direction of the y -axis. Compute

$$\iint_{\Sigma} \langle y, x + 1, z \rangle \cdot dS$$

Exercise 8 Let Σ be the portion of the cone $x = \sqrt{3y^2 + 3z^2}$ with $x \leq \frac{\sqrt{3}}{2}$, oriented in the direction of the x -axis. Compute

$$\iint_{\Sigma} \langle 2, 3x - 1, y + z \rangle \cdot dS$$