

## 1 Vector fields, curl, and divergence

**Exercise 1** Find the curl and divergence of the vector field

$$F(x, y) = \langle x, y \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 2** Find the curl and divergence of the vector field

$$F(x, y) = \langle y, x \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 3** Find the curl and divergence of the vector field

$$F(x, y) = \langle -y, x \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 4** Find the curl and divergence of the vector field

$$F(x, y) = \langle ye^{xy}, xe^{xy} \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 5** Find the curl and divergence of the vector field

$$F(x, y) = \langle \cos x - \sin(x)e^y - y^2, \cos(x)e^y - 2xy \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 6** Find the curl and divergence of the vector field

$$F(x, y, z) = \langle 2xyz, x^2z, x^2yz \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 7** Find the curl and divergence of the vector field

$$F(x, y, z) = \langle yz^2, xz^2, 2xyz \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 8** Find the curl and divergence of the vector field

$$F(x, y, z) = \langle e^x + z, \sin z - y, x + y \cos z \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 9** Find the curl and divergence of the vector field

$$F(x, y, z) = \langle 2 \cos x \cos y - 4e^z, -2 \sin x \sin y + 5z - y, 5y - 4xe^z + z^2 \rangle.$$

Determine if the vector field is conservative or not. If it is, find a potential function.

**Exercise 10** Find a vector field  $F(x, y)$  that has positive curl at the point  $(0, 0)$  and negative curl at the point  $(10, 0)$ .

**Exercise 11** Find a vector field  $F(x, y)$  that has positive divergence at the point  $(0, 0)$  and negative divergence at the point  $(0, 10)$ .

**Exercise 12** Find a vector field  $F(x, y)$  that has positive curl at the point  $(1, 1)$  and negative divergence at the point  $(-1, -1)$ .

**Exercise 13** Find a vector field  $F(x, y)$  with the property that:

- The flow-line that starts at the point  $(1, 0)$  passes through the point  $(2, 0)$  at a later time.
- The flow-line that starts at the point  $(0, 1)$  passes through the point  $(0, 2)$  at a later time.

**Exercise 14** Find a vector field  $F(x, y)$  with the property that:

- The flow-line that starts at the point  $(1, 0)$  passes through the point  $(2, 0)$  at a later time.
- The flow-line that starts at the point  $(0, 2)$  passes through the point  $(0, 1)$  at a later time.

**Exercise 15** Find a vector field  $F(x, y)$  with the property that:

- The flow-line that starts at the point  $(1, 0)$  passes through the point  $(0, 1)$  at a later time.
- The flow-line that starts at the point  $(2, 0)$  passes through the point  $(0, 2)$  at a later time.

**Exercise 16** Sketch a vector field  $F(x, y)$  with the property that:

- The flow-line that starts at the point  $(2, 0)$  is at the point  $(0, 1)$  after one unit of time.
- The flow-line that starts at the point  $(0, 2)$  is at the point  $(1, 0)$  after one unit of time.